

# In Praise of Unstable Fixed Points: The Way Things Actually Work

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## Abstract

In recent years a fashion has grown up to ascribe great importance to “quantum critical points” at  $T = 0$ , at the boundary between the basins of attraction to the stable fixed points of ordered ground states. I argue that more physical significance in connecting microscopic interactions with observed phenomena lies in the common phenomenon of partially ordered “liquid” states at higher temperatures, unstable phases which define the relevant degrees of freedom and may order in many different ways as the temperature is further lowered.

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In our field of strongly-correlated electronic phenomena, the first fad of the 21<sup>st</sup> century is the Quantum Critical Point.[1] (See Fig. 1.) This is defined as a point (along a line representing different values of some control parameter) where two ground states with different symmetries and different order parameters meet. The argument is that in the neighborhood of such a point we can show that the Ginzburg-Landau equations of the system have a higher symmetry and also have the scale-invariance appropriate to such a critical point. Any unusual behavior such as unexpected power laws and absence of conventional excitation spectra may be thus ascribed to the baleful influence of such a critical point. The “effective action” (I don’t know why such theories are always Lagrangian rather than Hamiltonian) which controls the low-energy behavior is restricted to being a functional of the appropriate “relevant variables”: the order parameters of the ordered phases. The world, then, is seen to be controlled “from the bottom up”: everything is described in terms of the low-energy degrees of freedom of the ground states, as in Fig. 1. The QCP is often mysterious, or hidden by the intrusion of yet another phase, but its influence expands as  $T$  is raised until its critical region encompasses the whole high  $T$  phase diagram. All relevant behavior is controlled by some ( $D+Z$ ) dimensional field theory; i.e., all we need to do is find what crank to turn on some universal dream machine.

As far as I know, there are no unequivocal examples of Fig. 1. Critical regions near  $T = 0$  are normally quite narrow.[2] But perhaps more important is the fundamental ontological question: What is the purpose of our scientific endeavor? If we abandon description of physics in terms of the microscopic systems and forces—electrons, ions, Coulomb interactions—then what is the point of our science? And this abandonment is precisely the goal of the QCP fad. The original purpose of the RNG in the study of critical behavior was to explain universality of exponents and yet retain the connection to microscopic

physics; but the QCP sect abandons the idea of the RNG as based on fundamental microscopic physics, ignoring the starting point of the RNG and dealing only with its end result. Microscopic variables are seen as “irrelevant” and “therefore” unimportant.

I would argue here that the physics of real condensed matter systems is quite otherwise. Fig. 1 is backwards. The spirit of the RNG is indeed correct: One wants to start at high energies and temperatures, and integrate out high frequency degrees of freedom in stages in order to build low-temperature models with manageable complexity and, hopefully, understandable behavior. But actual systems almost invariably fail to flow smoothly from high energy to  $T = 0$ . There is a strong tendency to hesitate in the neighborhood of an infrared-unstable fixed point, which is hopefully described by some effective “model” Hamiltonian which still has a large number of relevant degrees of freedom. This model is often the result of a projective transformation of the problem, removing certain high-energy degrees of freedom and replacing them with constraints. But it cannot be correctly viewed as a functional of the limited degrees of freedom of an order parameter with a point symmetry, no matter how complex. As we shall see, such phases are liquids, paramagnetic insulators, rare earth metals at normal temperatures, even the metallic Fermi liquid. I use the word “phase” advisedly because in many cases we recognize these as thermodynamically distinct phases—as e.g. the liquid state, which has no symmetry distinction from the gaseous state but is physically different.

So the world is more frequently described by a diagram like Fig. 2. One goes from microscopic physics via a projective transformation to an intermediate phase, which is not ordered but is already a renormalized description satisfactory in some intermediate range of energies and temperature—but is not a stable fixed point. At all but a few, specially tuned values of the system parameters, this phase evolves further into one of several different

ordered phases.

Let me illustrate this general structure in a number of cases of decreasing simplicity and familiarity. Fig. 3 shows the structure which is familiar to us in thousands of materials: molecular gases, organic and polymer systems, and the like. Starting from electrons and nuclei, one eliminates electronic degrees of freedom via the Born-Oppenheimer approximation, arriving at molecules with given intermolecular forces. (In relatively rare cases, perhaps arriving at a covalently bonded network such as amorphous silicon or  $SiO_2$ .) Thus one is left with no electronic degrees of freedom at all. In general, the attractive parts of these forces produce a dense phase without crystalline order, which at lower temperature orders, generally, into one of the hundreds of possible crystal structures. The only relevance to our further reasoning is to observe how meaningless it is to think of the liquid phase as arising out of quantum criticality at one or more of the (first-order) phase transitions between different crystal structures.

Fig. 4 illustrates the next simpler case: the Fermi liquid. Here the electronic degrees of freedom are not all gapped, so one has to eliminate the high-frequency ones via the Galivotti-Shankar renormalization[3] and one is left with the Landau Fermi liquid Hamiltonian plus quasiparticle scattering, for quasiparticles which are the residual degrees of freedom. (There are also, of course, lattice degrees of freedom—the phonons. Fortunately, Migdal’s theorem, shows that these can be treated perturbatively.)

But the Fermi liquid in general is not stable. It may further renormalize into not only  $L = 0$  BCS but any of the angular-dependent BCS states, depending on the nature of the residual interaction. It may also have SDW or ferromagnetic instabilities. Again, I would challenge any one to reconstruct the Fermi liquid as resulting from a QCP between any of these phases.

Now we come to a more complex case: “Mixed valence” (see Fig. 5). We start with actinide or lanthanide ions and metal electrons. We bypass the complications of the true “mixed valence” state, which renormalizes to a relatively simple metal (as shown by Haldane). That is, in Fig. 5 renormalization paths near the edge of the Anderson Model regime can flow into the “Kondo valence collapse” regime. But for reasonably symmetric systems renormalization is via a Schrieffer-Wolff transformation[4] to the “Kondo lattice.” This unstable fixed point is massively non-Fermi liquid: the lanthanide or actinide f electrons are represented by pure spins with no charge degrees of freedom, these latter having been projected away by the Anderson → Kondo renormalization.

But now we find that in many cases the Kondo Hamiltonian renormalizes again via the  $T_K$  mechanism to a new Fermi liquid! On the other hand, it may also happen that residual spin-spin interactions enforce ferromagnetism or antiferromagnetism. In a quite separate way, the Kondo *FL* is itself unstable to non BCS SC or to SDW or both. (Note that BCS does not happen: the spin-derived electrons do not interact strongly with phonons.)

Of course, I have been leading up to still another unstable fixed point: the RVB phase: a lightly-doped 2D Mott insulator. In this case the starting high-temperature Hamiltonian has strong local repulsion—a Hubbard model, with  $U \gg t$ —and the appropriate initial renormalization is the  $t - J$  Anderson-Rice transformation,[5] which leaves us with a truncated Hilbert space, containing hole and spin degrees of freedom and a  $J$  interaction between spins. As noted in a recent article, for low density the resulting phase renormalizes to a Fermi liquid, or, in 2D, to a Fermi liquid-like phase. (For low density, the Galitskii-Lee-Yang pseudopotential scheme is correct.) But for high density, a spin pseudogap phase may develop, which is distinct from the Fermi liquid and can condense into several different ordered phases: Néel antiferromagnet, d-wave superconductor, inhomogeneous stripe

phases, and possibly the staggered flux phase of Lee and Nayak et al.[6] The pseudogap phase is best described, probably, as having d-symmetry condensation of spinon pairs. It is not “caused” in any sense either by the SC-AF quantum critical point nor by the crossover to a true Fermi liquid(like) state. Fig. 6 is a sketch of the appropriate phase relationships. In conclusion, I would like to remark that none of the “liquid” phases (except possibly the Fermi liquid) is easy to analyze in any precise way—this has long been known for the true liquid, for instance, and we are only just learning it for the Mott paramagnet. This however does not imply that they have no identity or physical reality. The simplicity of BCS comes from the simplicity of the Fermi liquid state from which it arises; the difficulty of the calculation of melting points, in contrast, comes from the relatively mysterious nature of the liquid state. So we cannot expect high  $T_c$  cuprates to be simple; but we find that their physical motivation is not mysterious.

## References

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## Figure captions

Fig. 1 The Quantum Critical Point Picture. Above the QCP at  $T = 0$  is an Ever-expanding “Critical Region” of Anomalous Properties.

Fig. 2 The Intermediate Unstable Fixed Point Picture. Electrons and Ions Renormalize to an UFP, or Model, with Constrained Dynamics but  $\approx N$  Degrees of Freedom, Which Then Further Condenses to Various Ordered States.

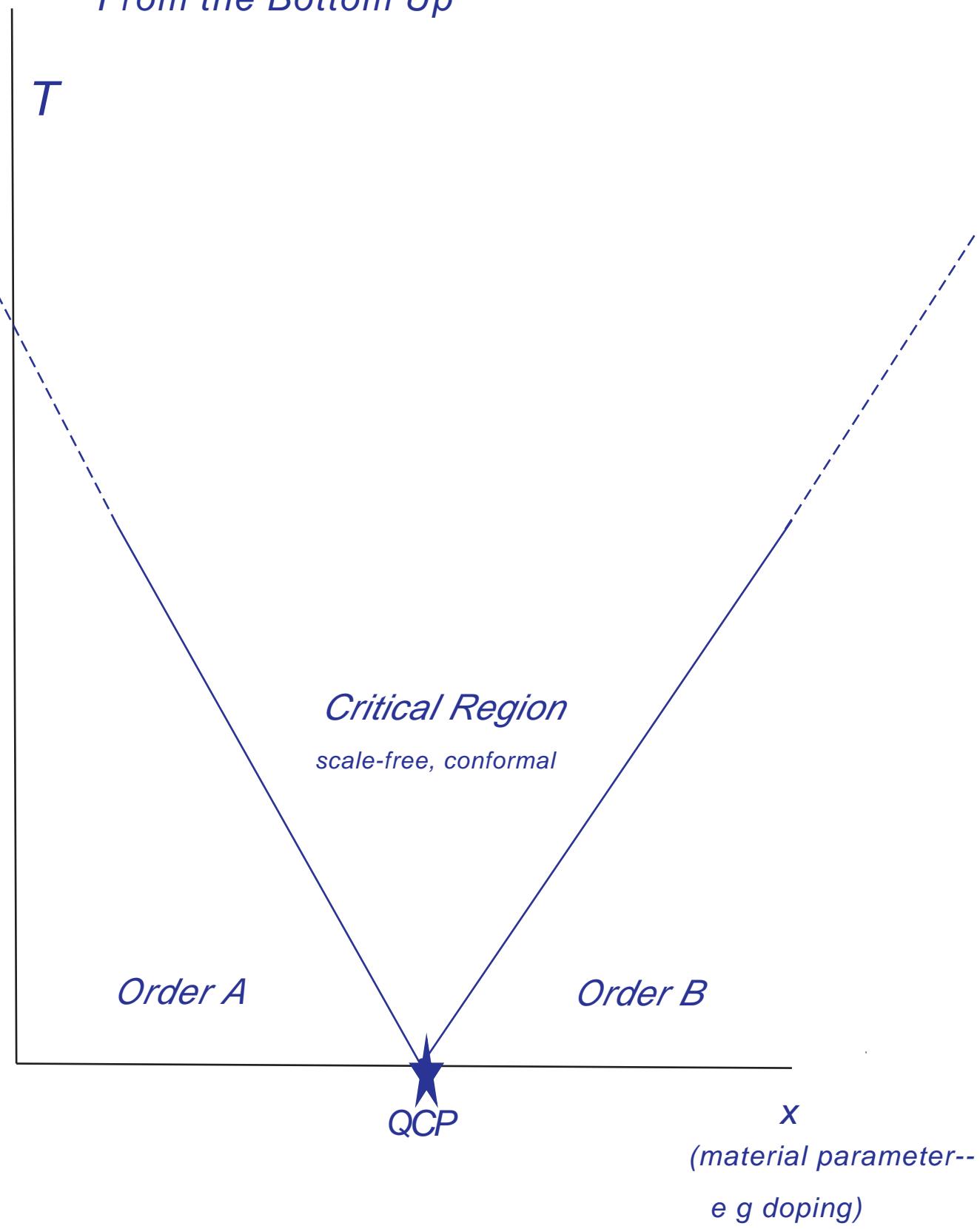
Fig. 3 The Molecular Liquid State as an Example of the UFP: The Born-Oppenheimer Projection Leaves Us with Molecular Motions Only. Molecular Liquid Can Condense into Many Phases.

Fig. 4 The Fermi Liquid as Example: Gallivotti-Shankar Projection Leaves Us with Quasiparticles Near a Fermi Surface, Which Then Can Condense into Ferromagnetism, Spin- or Charge-Density Waves or BCS with Any L.

Fig. 5 The Kondo Hamiltonian as an UFP. This Hamiltonian Has f-Electric Charge Fluctuations Completely Projected Out, Yet Can Renormalize to an Unusual Fermi Liquid, SDW, or  $L \neq 0$  BCS Superconductor.

Fig. 6 The RVB-Pseudogap State as an UFP, Only in the Low-Doping Regime of the 1-Band Hubbard Model. It Can Condense to Stripes, d-Wave Superconductor or Néel Antiferromagnet.

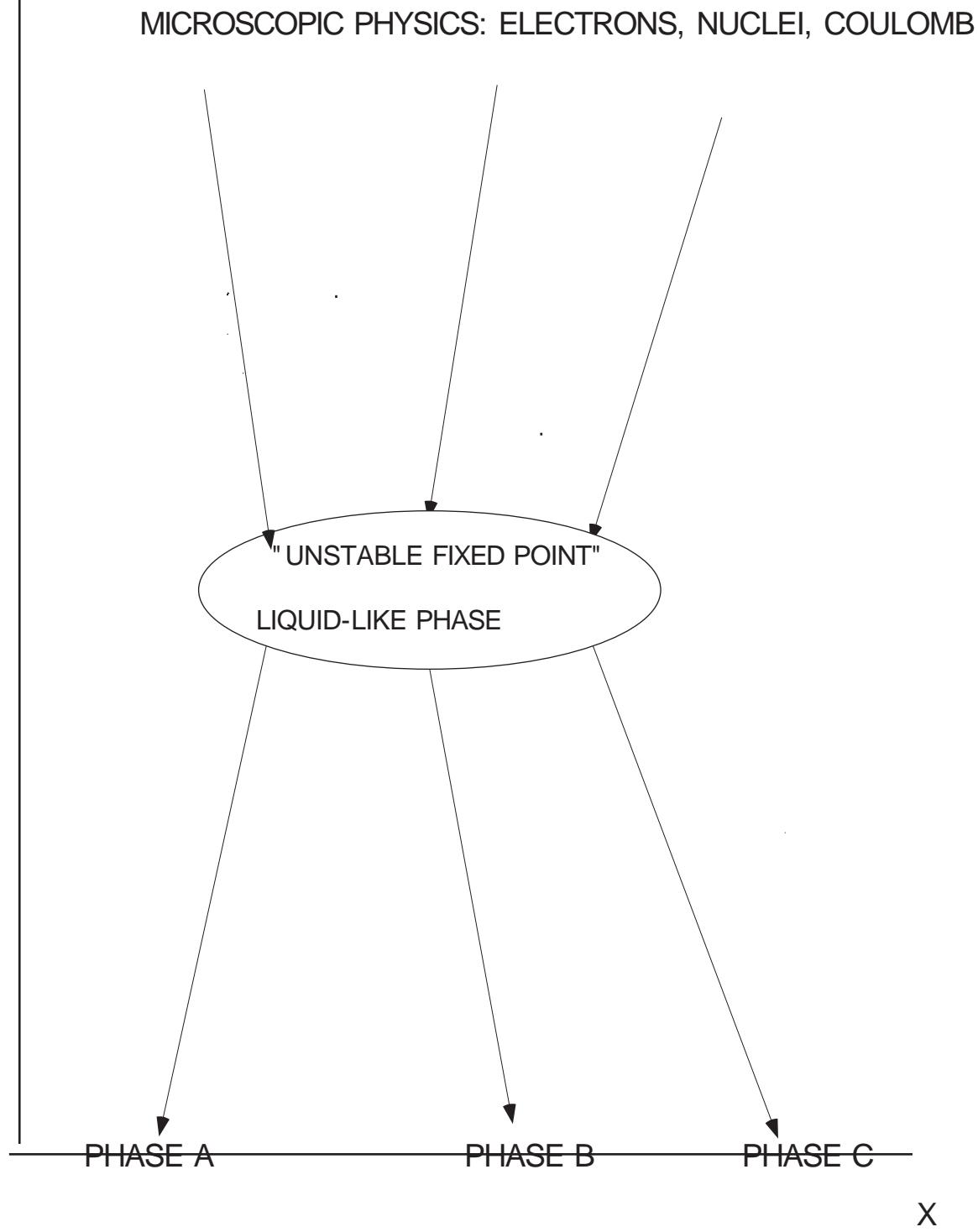
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From the Bottom Up*



# THE WAY THE WORLD R E A L L Y WORKS

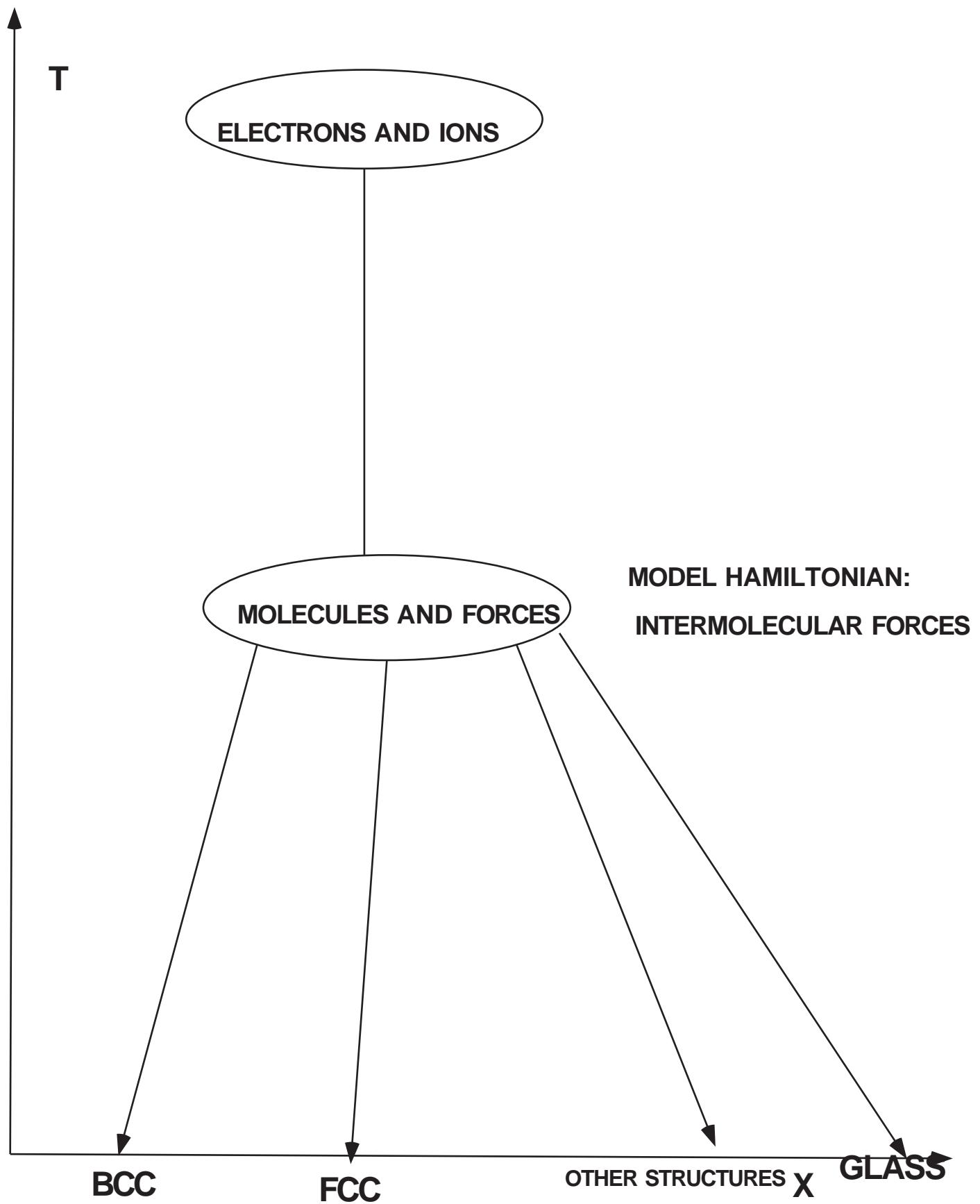
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MICROSCOPIC PHYSICS: ELECTRONS, NUCLEI, COULOMB



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## MOLECULAR LIQUIDS AND SOLIDS



# THE FERMI LIQUID

